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RESISTANCE OF A SPHERE IN A SLOW FLOW OF A VISCOELASTIC FLUID

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The authors have obtained an approximate solution of the problem of the resistance of a rigid sphere in a slow flow of a Maxwell viscoelastic fluid that is in good agreement with experimental data [1] for Weissenberg numbers We ≤ 0.7. It is shown that the effect of a decrease in the coefficient of resistance of a sphere in the interval 0.1 ≤ We ≤ 0.7 established experimentally is determined in full measure by the linear viscoelastic properties of the Maxwell fluid.

In [1], the sedimentation of single spherical particles in solutions of Separan AR-30 polyacrylamide in a glucose syrup was investigated for Reynolds numbers $1.69 \cdot 10^{-5} \le \text{Re} \le 8.1 \cdot 10^{-2}$ and Weissenberg numbers $1.66 \cdot 10^{-4} \le \text{We} \le 2.02$, where $\text{Re} = 2V_\infty r\rho/\eta$ and $\text{We} = V_\infty \lambda/r$. It was shown that the experimental values of the coefficient of resistance of a sphere obey the dependence

$$C_f = 24\text{Re}^{-1}\,\text{Xe}\,,$$

where the parameter Xe ≈ 1 for We < 0.1, while in the interval 0.1 ≤ We ≤ 0.7, Xe decreases monotonically, attaining the constant value Xe ≈ 0.74 for We > 0.7.

For the experiment, we selected fluids that have nearly linear viscoelastic properties. However, as the authors of [1] note, the existing theoretical solutions of the problem of the resistance of a sphere in a flow of a linear viscoelastic fluid do not predict any significant effect of a decrease in the coefficient of resistance. Thus, in the experiment with ReWe = 0.0498 and We = 1.817, Xe ≈ 0.74 is obtained, while according to the calculations of the authors of [1], the known analysis of Altman and Denn [2] predicts the minimum value Xe = 0.958 for ReWe ≤ 0.05. On this basis, in [1], the conclusion of the impossibility of obtaining an adequate solution without using the theory of nonlinear viscoelasticity is made.

Employing an approximate method of analysis, we show that the presented experimental results [1] can be described rather accurately within the framework of the theory of linear viscoelasticity for We ≤ 0.7.

We consider a slow steady-state unbounded flow of a Maxwell viscoelastic fluid about a rigid sphere. By the steadiness of the flow we will mean the invariability of the field of fluid velocities with time.

Directing the principal axis of the spherical system of coordinates (R, θ, φ) with the pole at the center of the sphere parallel to the vector of the uniform-flow velocity at infinity V_∞ and assuming by virtue of the axial symmetry of the flow ($V_\varphi = 0$) and the incompressibility of the fluid the existence of a Stokes stream function, we write the expression for it known for the case of flow about a sphere [3]:

$$\psi = \frac{1}{2} V_\infty R^2 \sin^2\theta \left[\frac{1}{2}\left(\frac{r}{R}\right)^3 - \frac{3}{2}\left(\frac{r}{R}\right) + 1\right], \quad 0 \le \theta \le \pi\,. \tag{1}$$

From Eq. (1) we find expressions for the components of the fluid velocity as

$$V_R = \frac{1}{R^2 \sin\theta} \frac{\partial \psi}{\partial \theta} = \frac{1}{2} V_\infty \cos\theta \left[\left(\frac{r}{R}\right)^3 - 3\left(\frac{r}{R}\right) + 2\right], \tag{2}$$

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$$V_\theta = -\frac{1}{R\sin\theta}\frac{\partial\psi}{\partial R} = \frac{1}{4}V_\infty \sin\theta\left[\left(\frac{r}{R}\right)^3 + 3\left(\frac{r}{R}\right) - 4\right]. \tag{3}$$

Using (2)and (3), we find the tangential component of the tensor of the rates of deformation on the surface of the sphere [4]

$$\dot{\gamma}(\theta) = \left(\frac{\partial V_\theta}{\partial R} + \frac{1}{R}\frac{\partial V_R}{\partial\theta} - \frac{V_\theta}{R}\right)_{R=r} = \left(\frac{\partial V_\theta}{\partial R}\right)_{R=r} = -\frac{3V_\infty}{2r}\sin\theta. \tag{4}$$

By virtue of the steadiness of the velocity field the trajectory of the motion of the fluid particles coincides everywhere with the streamlines [4]. This enables us, considering the time of particle flow about the sphere to be a parameter, to write Eq. (4) in the form

$$\dot{\gamma}(t) = -\frac{3V_\infty}{2r}\sin(\omega t), \tag{5}$$

$$\theta = \omega t, \quad 0 \le \omega t \le \pi. \tag{6}$$

Using the formal validity of arbitrarily prescribing function (6), we will consider ω = const as a first approximation. Then the expression for ω is found from the condition $\omega T = \pi$, where T is the time during which a fluid particle experiences deformation in flow about the sphere. The magnitude of T cannot be established analytically and must be determined from the condition of the best approximation of the sought solution of the problem to experiment.

Writing the rheological equation of state of the Maxwell fluid for the tangential component of the stress tensor

$$\tau + \lambda\frac{d\tau}{dt} = \eta\dot{\gamma},$$

we find under the initial condition $\tau(0) = 0$

$$\tau(t) = \frac{\eta}{\lambda}\exp\left(-\frac{t}{\lambda}\right)\int_0^t \dot{\gamma}(t)\exp\left(\frac{t}{\lambda}\right)dt. \tag{7}$$

Substituting expression (5) into solution (7) and computing the integral, we obtain

$$\tau(t) = \frac{3\eta\,\omega\,\omega_1}{\pi(\omega^2 + \omega_1^2)}\left[\omega\cos(\omega t) - \omega_1\sin(\omega t) - \omega\exp(-\omega_1 t)\right]$$

or

$$\tau(\theta) = \frac{3\eta\,\omega\,\omega_1}{\pi(\omega^2 + \omega_1^2)}\left[\omega\cos\theta - \omega_1\sin\theta - \omega\exp\left(-\frac{\omega_1}{\omega}\theta\right)\right], \tag{8}$$

where $\omega_1 = 1/\lambda$.

Using expressions (5) and (8) and considering the Maxwell fluid to be a viscous fluid with the effective viscosity $\mu(\theta = \tau(\theta)/\dot{\gamma}(\theta)$, we adopt the pressure distribution in the flow disturbed by the sphere by analogy with the Stokes solution [4]

$$P(R,\theta) = -\frac{3}{2}\mu r V_\infty\frac{\cos\theta}{R^2} + P_\infty,$$

for $R = r$

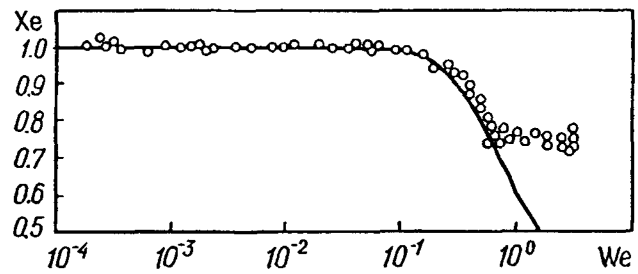

Fig. 1. Data of the experiment [1] and theoretical dependence (13) of Xe on We (Xe and We are dimensionless).

$$P(\theta) = -\frac{3}{2}\mu V_\infty \frac{\cos\theta}{r} + P_\infty . \tag{9}$$

Multiplying $\tau(\theta)$ and $P(\theta)$ by the area of the elementary annular surface of the sphere $2\pi r^2 \sin\theta d\theta$ and projecting the elementary forces obtained onto the direction of the velocity V_∞, we write the force of resistance of the sphere to the incoming flow in the form [4]

$$F = \int_0^\pi (-\tau(\theta)\sin\theta - P(\theta)\cos\theta)\, 2\pi r^2 \sin\theta d\theta . \tag{10}$$

Substituting expressions (8) and (9) into Eq. (10), we obtain

$$F = 6\pi\eta r V_\infty \frac{1 + 0.5(\omega/\omega_1)^2 [1 - \exp(-\pi\omega_1/\omega)]}{1 + (\omega/\omega_1)^2} . \tag{11}$$

Taking the time of deformation of a fluid particle to be $T = 2r/V_\infty$, we have

$$\omega = \frac{\pi V_\infty}{2r} . \tag{12}$$

Then, allowing for the fact that $\mathrm{We} = \lambda V_\infty / r = 2\omega/\pi\omega_1$, from (11) we obtain the following dependence of the force of resistance of the sphere on the Weissenberg number:

$$F(\mathrm{We}) = 6\pi\eta r V_\infty \frac{1 + 0.5(\pi\,\mathrm{We}/2)^2 [1 - \exp(-2/\mathrm{We})]}{1 + (\pi\,\mathrm{We}/2)^2} .$$

We find the coefficient of resistance of the sphere as

$$C_f = \frac{F(\mathrm{We})}{\frac{1}{2}\rho V_\infty^2 \pi r^2} = \frac{24}{\mathrm{Re}} \mathrm{Xe}(\mathrm{We}), \quad \mathrm{Xe}(\mathrm{We}) = \frac{1 + 0.5(\pi\,\mathrm{We}/2)^2 [1 - \exp(-2/\mathrm{We})]}{1 + (\pi\,\mathrm{We}/2)^2} . \tag{13}$$

The curve calculated by formula (13) describes well the experimental data with $\mathrm{We} \le 0.7$ (see Fig. 1). The choice of expressions for ω that are not the same as (12) affects the accuracy of describing the experimental data by the theoretical curve in the interval $0 \le \mathrm{We} \le 0.7$ but does not make it possible to obtain its flat branch for $\mathrm{We} > 0.7$ (see Fig. 1). This means that the assumption of a Stokes distribution of velocities and pressure in the sphere-disturbed flow adopted in this work is, apparently, valid only for $\mathrm{We} \le 0.7$.

In summary, we note that, within the above limits of applicability, the solution found describes in full measure the decrease in the coefficient of resistance of a sphere in the interval $0.1 < \mathrm{We} < 0.7$ and establishes the effect as being due to the linear viscoelastic properties of a Maxwell fluid.

NOTATION

V_∞, velocity of the flow at infinity (the velocity of the sphere in the motionless fluid); γ, sphere radius; ρ, fluid density; η, viscosity; λ, relaxation time; $\dot{\gamma}$, rate of deformation; t, time; ω, angular velocity of the polar radius of a fluid particle; τ, tangential stress.

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